Adjoint-Based Aerodynamic Shape Optimization Using a Spline-Based Parametric Geometry Framework

Harsh Menon

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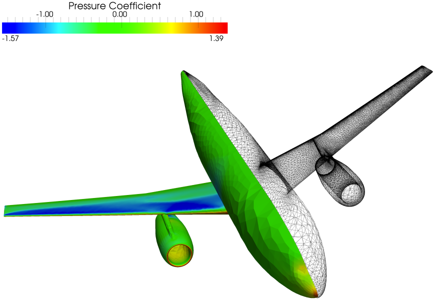
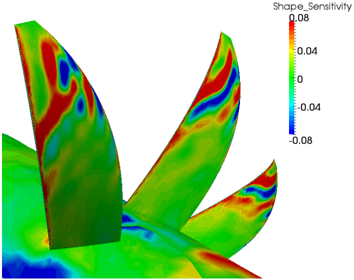
Overview

- Introduction
  - High-fidelity aircraft design
  - Optimization Methods
  - Sensitivity Analysis Methods
- Adjoint-based Optimization
  - Adjoint Equations
  - Components of a Design Cycle
  - Geometry Parameterization
  - Mesh Sensitivities (Embedded vs Boundary Conforming)
- Parametric Geometry Framework
  - Goals
  - Validation Case: Incidence Optimization of an airfoil
  - Propeller Optimization: Incidence, Chord, and Both
  - Extensions to Fuselage-Wing Configurations

High-fidelity aircraft design

- Begin with a baseline configuration that has been designed using some low-fidelity design tools (panel methods)
- Need high-fidelity methods in cases where lower-fidelity tools are not adequate (predicting rotor performance in hover) or where a high dimensionality is required (to smooth shock waves in transonic flow)
- Becomes computationally expensive as the design process usually involves a large number of variables
- Have been used successfully to design aircraft/components for aero, aero-structural and aero-acoustic objective functions





Optimization Methods

- Intuition - decreases with increasing dimensionality
- Grid or Random Search - cost becomes impractical with increasing dimensionality
- Genetic Algorithms - good for discrete design variables and very robust but infeasible for a large number of design variables
- Nonlinear Simplex - simple and efficient but inefficient for more than a few variables
- Response Surfaces - requires a large number of function evaluations to create fit ($(N+1)(N+2)/2$ for a quadratic response surface where N is the number of variables)
- Gradient Based - most efficient for problems with a large number of design variables; assumes smoothness in the objective and constraints

Sensitivity Analysis Methods

- Sensitivity analysis could be a potential bottleneck for gradient based optimization
- Finite Differences - easy to implement, susceptible to subtractive cancellation errors, requires N function evaluations where N is the number of variables

$$f'(x) = \frac{f(x+h) - f(x)}{h} + \mathcal{O}(h)$$

- Complex-Step Derivative - robust and accurate, requires N function evaluations

$$f'(x) = \frac{Imag(f(x+ih))}{h} + \mathcal{O}(h^2)$$

- Algorithmic / Automatic Differentiation - accurate, easy to implement, cost varies
- Analytic Methods - long development time, cost can be independent of N
- Adjoint Method - accurate, cost virtually independent of the number of design variables

Adjoint Equations

Given a set of design variables D, we want to minimize a scalar objective function J, subject to the constraint that it satisfies the discrete flow equations and boundary conditions R,

$$\begin{aligned} \underset{D}{\text{minimize}} \quad & J(Q, D, M) \\ \text{subject to} \quad & R(Q, D, M) = 0 \end{aligned}$$

where Q is the state vector, D is the vector of design variables and M represents the mesh. We further state that the mesh M is a function of the triangulation T which in turn depends on D. [M = f(T(D))]

Adjoint Equations

We use the above equations to form the Lagrangian

$$L(Q, D, M) = J(Q, D, M) - \psi^T R(Q, D, M)$$

Taking perturbations in Q,D and M we get

$$dL = \left(\frac{\partial J}{\partial Q} - \psi^T \frac{\partial R}{\partial Q}\right) dQ + \left(\frac{\partial J}{\partial D} - \psi^T \frac{\partial R}{\partial D}\right) dD + \left(\frac{\partial J}{\partial M} - \psi^T \frac{\partial R}{\partial M}\right) dM$$

Using the relationship between the mesh and the triangulation we get

$$dL = \left(\frac{\partial J}{\partial Q} - \psi^T \frac{\partial R}{\partial Q}\right) dQ + \left(\frac{\partial J}{\partial D} + \frac{\partial J}{\partial M}\frac{\partial M}{\partial T}\frac{\partial T}{\partial D} - \psi^T \left[\frac{\partial R}{\partial D} + \frac{\partial R}{\partial M}\frac{\partial M}{\partial T}\frac{\partial T}{\partial D}\right]\right) dD$$

If we choose ψ to obey the adjoint equation

$$\frac{\partial J}{\partial Q} = \psi^T \frac{\partial R}{\partial Q}$$

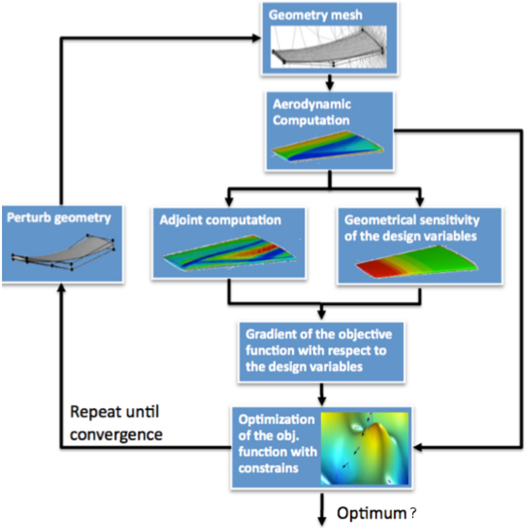
Adjoint Equations

we obtain

$$dL = \left(\frac{\partial J}{\partial D} + \frac{\partial J}{\partial M} \frac{\partial M}{\partial T} \frac{\partial T}{\partial D} - \psi^T \left[\frac{\partial R}{\partial D} + \frac{\partial R}{\partial M} \frac{\partial M}{\partial T} \frac{\partial T}{\partial D} \right] \right) dD$$

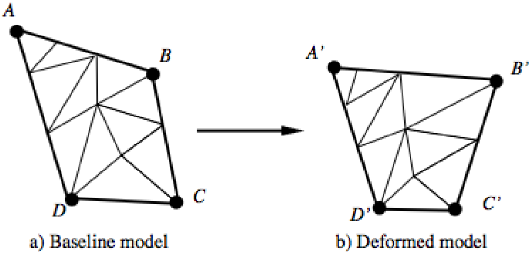
- The advantage of the method lies in the fact that the above equation is independent of changes in the state vector (Q) and hence no additional flow solutions are required. Thus, in order to evaluate the sensitivities of N variables to a given cost function, we need to solve the adjoint equation which is approximately computationally equivalent to one flow solution
- Method becomes less attractive as the number of constraints becomes greater than the number of variables since each constraint requires an additional flow solve

Components of a Design Cycle



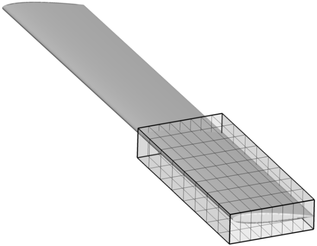
Geometry Parameterizations

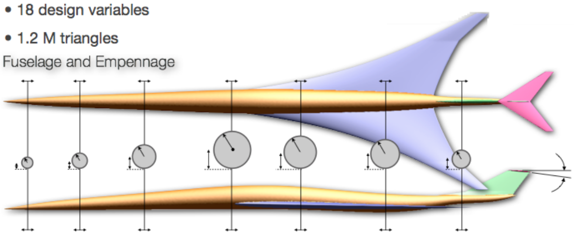
- Discrete Mesh Points - every point on the surface mesh is a variable, easy to implement but difficult to enforce smoothness; shapes might not be manufacturable
- CAD-based Approach - has the ability to model complex geometry and hence ideal for parametric changes in geometry; computation of sensitivities not trivial
- Domain-Element Formulation - groups a bunch of surface mesh points into a domain element; as nodes of the domain element move the mesh points in the element move as well based on an inverse mapping; applicable only for simple geometries



Geometry Parameterizations

- PDE-Based Approach - view the surface mesh as the solution of a PDE; hard to parameterize geometry and can be computationally expensive
- Free-form Deformation Boxes (FFD) - based on SOA algorithms; enclose the surface mesh of interest with a box and deform the mesh by varying the control points of the box; robust, applicable to complex geometries; hard to get physical intuition into what the moving the control points does to the surface mesh
- Parametric Approach Based on Using Polynomials and Splines - reduces the number of design variables, can compute analytic sensitivities and have physical understanding of parameters; harder to create complex geometries



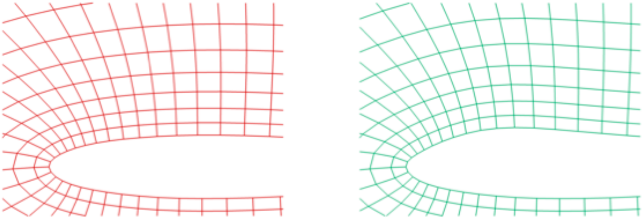


Mesh Sensitivities

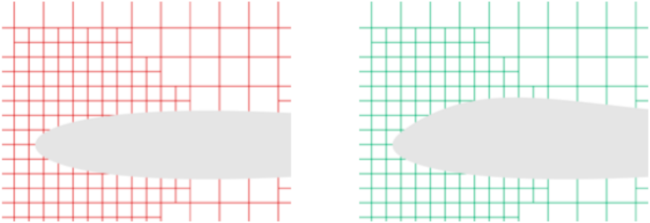
- As the geometry changes, how does the surface / volume mesh change?
- Can either regenerate the mesh or deform the mesh
- Regeneration allows for robustness, can leverage existing grid generation codes, but not smooth
- Deformation is smooth, but often not robust to large changes in design variables
- Contrast differences between the two by looking at embedded boundary methods (regeneration) and boundary-conforming methods (deformation) that have been used successfully for adjoint-based optimization

Mesh Sensitivities (Embedded vs Boundary-Conforming)

- Domain mapping formulation (body-fitted methods)



- Embedded-boundary formulation (cut-cell)



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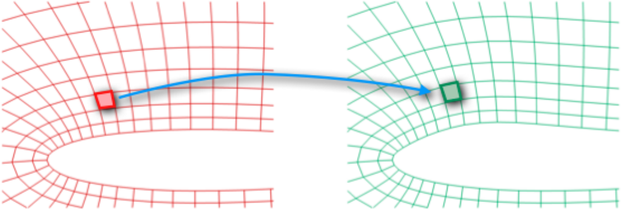

$$\frac{\partial \mathbf{M}}{\partial \mathbf{T}} \neq 0$$

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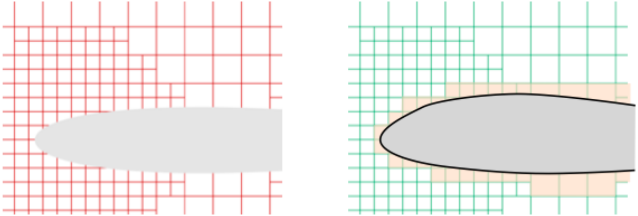


$$\frac{\partial \mathbf{M}}{\partial \mathbf{T}} = 0$$

$$\frac{\partial \mathbf{M}}{\partial \mathbf{T}} \neq 0$$

Mesh Sensitivites (Embedded vs Boundary-Conforming)

- Can we use regeneration exclusively in a boundary-conforming mesh?
- Answer depends on the following questions
  - If you change the geometry from A to B, will the mesh for A change to the mesh for B smoothly?
  - Are the changes in the geometry proportional to the changes in the mesh? (Will a small perturbation in the geometry produce a small perturbation in the mesh?)

Parametric Geometry Framework

Goals

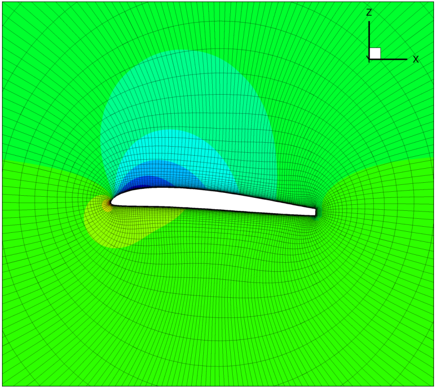
- Be able to generate 2D and 3D geometries with ease using physically meaningful parameters
- Be able to interface with legacy codes to leverage their adjoint capabilities
- Be able to provide surface mesh sensitivities
- Be robust to large changes in parameters
- Be able to deform the surface mesh

Validation Case: Incidence Optimization of an Airfoil

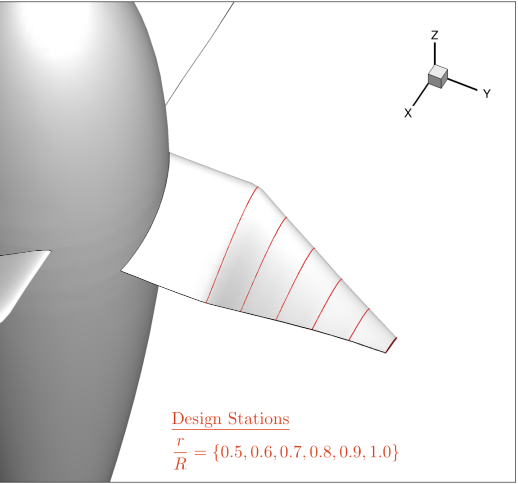
- 2-D Single Variable Unconstrained Optimization Problem
- Goal was to minimize

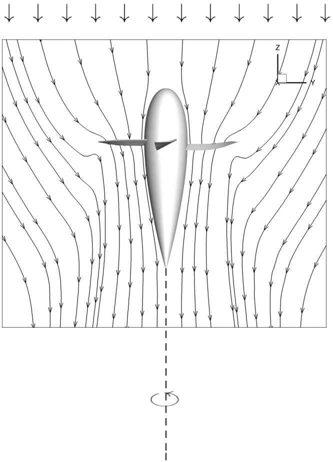
$$C_d + 100(C_l - C_{l,target})^2 \qquad (1)$$

- Airfoil incidence was the only parameter and was bounded to lie between $\pm 8^\circ$
- Starting Incidence of 0°
- SNOPT used as the optimizer
- Precomputed solution by generating polars - agreed with result from SNOPT
- Validated gradients and analytic mesh sensitivities



Design of Propellers



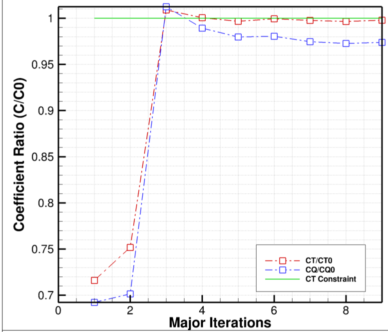


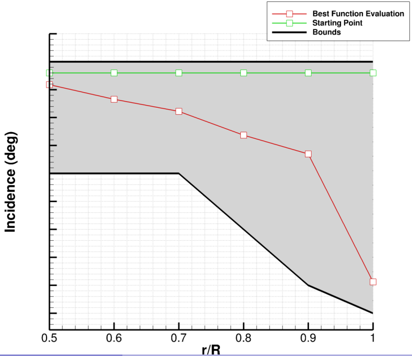
Incidence Optimization

- The goal was to

$$\begin{array}{ll} \underset{x}{\text{minimize}} & C_Q \\ \text{subject to} & C_T \geq C_{T,target} \end{array}$$

- Incidence at the 6 design stations were variables
- Starting point was a constant incidence distribution
- SNOPT used as the optimizer



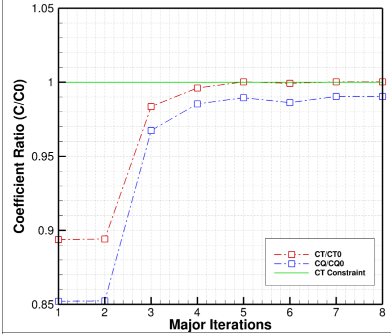


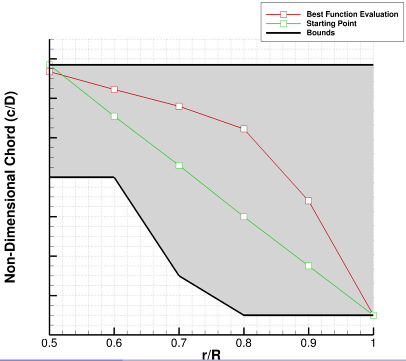
Chord Optimization

- The goal was to

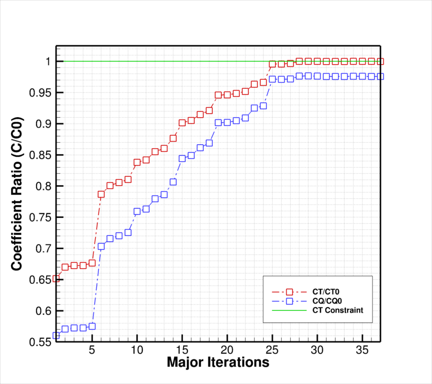
$$\begin{array}{ll} \underset{x}{\text{minimize}} & C_Q \\ \text{subject to} & C_T \geq C_{T,target} \end{array}$$

- Chord at the 6 design stations were variables
- Starting chord distribution was a linearly decreasing chord
- SNOPT used as the optimizer





Incidence & Chord



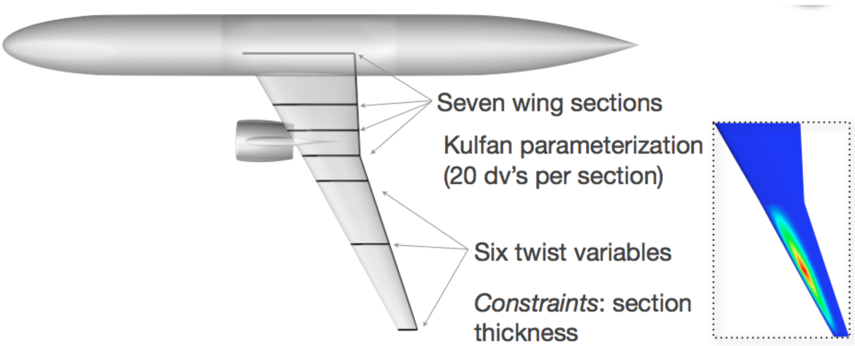
- The goal was to

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & C_Q \\ \text{subject to} \quad & C_T \geq C_{T,target} \end{aligned}$$

- Incidence and Chord at the 6 design stations were variables
- Starting point was a constant incidence and linearly decreasing chord distribution
- SNOPT used as the optimizer

Design of Fuselage-Wing Configurations

- Current work focused on extending framework to design fuselage-wing configurations



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